If you are using a Theorem or a Result proved in class then please state it clearly.

1. Let $s_1 = 3$. Define $s_{n+1} := \frac{1}{2}(s_n + \frac{5}{s_n})$ for $n \in \mathbb{N}$. Show that $\lim_{n\to\infty} s_n$ exists and identify the limit.

2. Let $E : \mathbb{R} \to \mathbb{R}$ be such that $E(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. Show, from first principles, that
$$\lim_{h\to 0} \frac{E(h) - 1}{h} = 1.$$

3. Please decide whether the following statements are true or false. If you decide that a statement is true then please provide a proof. If you decide that a statement is false then please provide a counter-example with justification.
   (a) The sequence $\{x_n\}_{n=1}^{\infty}$ given by $x_n = \sum_{k=1}^{n^2} \frac{1}{\sqrt{n^4+k}}$ converges.
   (b) Let $f : [0, 1] \to [0, 1]$ be a continuous function. There exists an $x \in [0, 1]$ such that $f(x) = x$.
   (c) Suppose $\{x_n\}_{n=1}^{\infty}$ is a sequence of non-zero real numbers such that $\limsup_{n\to\infty} \left|\frac{x_{n+1}}{x_n}\right| \geq 1$. Then $\sum_{n=1}^{\infty} x_n$ is divergent.
   (d) Let A be a bounded subset of $\mathbb{R}$ and $\alpha = \sup(A)$. Then α is a limit point of A.
   (e) Suppose $f : (0, 1) \to \mathbb{R}$ be differentiable and $f' : (0, 1) \to \mathbb{R}$ is bounded. Then f is uniformly continuous.

4. A function $f : \mathbb{R} \to \mathbb{R}$ is called upper-semicontinuous if $\forall x \in \mathbb{R}$ and $\forall \epsilon > 0, \exists \delta \equiv \delta(x, \epsilon) > 0$ such that $y \in (x - \epsilon, x + \epsilon)$ implies $f(y) < f(x) + \epsilon$. Show that f is upper-semicontinuous if
$$x_n \to x \in \mathbb{R} \Rightarrow \limsup_{n\to\infty} f(x_n) \leq f(x).$$
   Give an example of a function that is upper-semicontinuous but not continuous.

5. If $x \in [0, 1]$ and $n \in \mathbb{N}$, show that
$$|\ln(1 + x) - (x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots + (-1)^{n-1}\frac{x^n}{n})| < \frac{x^{n+1}}{n+1}$$